Intro to Credences

Belief and its Limits (BaiL); Seminar 3 ANU
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I. Opening Task

For discussion in this class, I'd like to introduce a stricter hand-finger system. You know the drill!

Suppose that our levels of confidence—our "credences"—in claims can be measured on the unit interval $[0,1]$; 1 for absolute certainty and 0 for absolute certainty in the negation.

So, e.g. $c(p) = 0.2$ says your level of confidence in claim p is .2.

According to Subjective Bayesiansim, your credences are rational just in case:

1. If p is a tautology, $c(p) = 1$.
2. If p and q are mutually exclusive, then $c(p \vee q) = c(p) + c(q)$.

One cool thing about Bayesianism is that *quite a lot* follows from just these two constraints. See chapter 2 of Titlebaum for a list.

Questions: Are you a Subjective Bayesian? That is do you think all and only credences satisfying these constraints are rational?

Or, do you think some credences that satisfy these constraints aren't rational? Or perhaps that some credences are rational even if they don't satisfy these constraints?

Is there anything else about the Bayesian picture you find weird?

Here's one weird upshot of this minimal Bayesian picture. Consider

> **Detective.** A homicide detective has found only two aspects, Alice and Bob, who could have committed the murder. He reveals that at this stage he is at least somewhat more inclined to think that Alice is the guilty one. "Why?" We ask. "No reason," he replies. "Unfortunately at this stage I have no evidence to go on."

Case is from (White, 2009, p.172)

Suppose the detective is .6 confident Alice did it, and .4 confident Bob did it. So his credences are perfectly in line with Subjective Bayesianism. But surely the detective is doing *something* wrong...

I'm idealizing and supposing that he's certain either Alice or Bob who it.

II. The Principle of Indifference

It seems like the detective is going wrong as, since he has no more reason to suppose Alice did it than Bob, he should assign *equal* credence to them having done it.

Say that p and q are **evidentially symmetrical**, $p \approx q$, for a subject just in case their evidence no more supports one than the other. Then more generally we might add a further constraint:

POI: If $p \approx q$, then $c(p) = c(q)$.

Which entails:

POI*: If $\{p_1, p_2, ..., p_n\}$ is a partition for you such that $p_1 \approx p_2 \approx ... \approx p_n$, then for all i, $c(p_i) = \frac{1}{n}$.

$\{p_1, p_2, ..., p_n\}$ is a partition for you means that you're certain of their disjunction and each p_i is mutually exclusive.

Importantly:

> POI... puts a *normative* constraint on what your *credence* may be. It entails that in a position of ignorance you are not rationally permitted to be more confidence of one proposition than another. It is not to be confused with a principle for determining what the *objective probabilities* or *chances* are.

II. The Mystery Square

There is a very famous problem for POI. Here is White's version:

Original from Joseph Bertrand. It's sometimes called "Bertand's Paradox". Discussed in detail by van Fraassen in *Laws and Symmetry*.

> *Mystery Square.* A mystery square is known only to be no more than *two* feet wide. Apart from this constraint, you have no relevant information concerning its dimensions. What is your credence that it is less than *one* foot wide?

(i) You have no more reason to assume the square is less than 1 foot wide than it is more than 1 foot wide: $L_1 \approx L_2$

$L_1 : 0 \leq \text{length} < 1$ ft.
$L_1 : 1 \leq \text{length} \leq 2$ ft.

(ii) So, by POI: $c(L_1) = c(L_2) = \frac{1}{2}$.

(iii) You have no more reason to assume it's area is less than 1 square foot, than it is between 1 and 2 square feet, or 2 and 3, or 3 and 4: $A_1 \approx A_2 \approx A_3 \approx A_4$.

$A_1 : 0 \leq \text{area} < 1$ sq. ft.
$A_2 : 1 \leq \text{area} < 2$ sq. ft.
$A_3 : 2 \leq \text{area} < 3$ sq. ft.
$A_4 : 3 \leq \text{area} \leq 4$ sq. ft.

(iv) So, by POI: $c(A_1) = c(A_2) = c(A_3) = c(A_4) = \frac{1}{4}$

(v) L_1 if and only if A_1. So $P(L_1) = P(A_1)$; contradiction.

(ii) and (iv) rely on POI. Since that seems to be the only uncontroversial premise, seems as though we need to deny POI...?

Reaction 1: Deny POI

Suppose we *do* deny POI here. Then what credences *should* we assign these propositions like L_1? Does anything go? Am I, say, allowed to be completely *certain* of L_1? But if that assignment is irrational, why?

Liu (2025) suggests we endorse "Permissivism" and endorse a partition-sensitive restriction of POI. Permissivists reject:

(Uniqueness) A single body of evidence always uniquely determines a fully rational credence function.

See Feldman (2010) for pioneering discussion. There is a HUGE literature on whether Uniqueness holds.

Liu suggests that one may choose between either partitioning the space through length—and being indifferent over it—or partitioning the space over area—and being indifferent over that. But this just pushes the question back: which *partitions* are we permitted to be indifferent over? Surely not all of them!

Consider the partition: $\{L_1, A_2 \vee A_3, A_4\}$. It would be bizarre to be indifferent over this partition. But what could rule it out?

Another option is to deny an even more fundamental assumption in Bayesian epistemology: that one should always assign a single-value probability to each proposition... we'll consider this move later.

Reaction 2: Deny something else!

White (2009) thinks something must be up with, not POI, but rather (i) or (iii). Restated:

(i) $L_1 \approx L_2$.

(iii) $A_1 \approx A_2 \approx A_3 \approx A_4$.

White argues these give rise to paradox even without considering POI. We just need two more principles:

(Transitivity) If $p \approx q$ and $q \approx r$ then $p \approx r$.

(Equivalence) If p and q are known to be equivalent, then $p \approx q$.

With these, we get another paradox:

(vi) $A_2 \approx A_1$ (By (iii))

I am also assuming something like a symmetry principle here: $p \approx q$ iff $q \approx p$.

(vii) $A_1 \approx L_1$ (By Equivalence)

(viii) $L_1 \approx L_2$ ((i), repeated)

(ix) $L_2 \approx A_2 \vee A_3 \vee A_4$ (By Equivalence)

(x) $A_2 \approx A_2 \vee A_3 \vee A_4$ (Transitivity with (vi), (vii), (viii), (ix))

(x) is absurd: *of course* we have more reason to think that the square is between 2 and 4 square feet than that it is between 2 and 3 square feet. $A_2 \vee A_3 \vee A_4$ is true in all of the circumstances A_2 is *and more*.

We have arrived at (x) without using POI. And Equivalence and Transitivity both seem extremely plausible. So perhaps it's (i) or (iii) that need to be given up. — But these were used in the argument against POI! So, White claims, that argument has been undermined.

Okay... but where exactly did we go wrong? Should we deny (i) or (iii). If so, what is my reason for thinking L_1 is more likely than L_2, or perhaps that A_1 is more likely than one of $A_2 - A_4$ (or vice versa)?

White says...

> Well, okay, so I don't really have an answer. Part of what is puzzling here stems from the temptation to think that my reasons or evidence must be transparent to me... These reasons [that either undermine (i) or (iii)] seem to be rather mysterious, accessible if at all only to enlightened souls... Suffice it to say that there are reasons to resist the temptation to think that your reasons or evidence must always be known to you.

He then cites Williamson (2000), who famously argues you are not always in a position to know what your evidence is.

Are we satisfied with this response...?

III. Imprecise Credences

There's a different kind of reaction to these problems for POI. Perhaps, when there is no evidence bearing on whether p, we should not assign a precise, middling credence to p, but rather spread our credence across a range of values. Our credence should be "imprecise" or "mushy".

For example, consider two urns:

> **Urn 1.** You know it contains 50 blue balls and 50 green balls.
>
> **Urn 2.** You know it contains some mixture of green/blue balls, but you have no idea what mixture.

Less controversially, your credence that you randomly select a green ball from Urn 1 should be 50%.

What should your credence be that you randomly select a green ball from **Urn 2**? Some people in favor of imprecise credences—e.g. Joyce (2005)—think you credence should be the interval $[0, 1]$.

Perhaps, in Mystery Square, your credences should also take the form of an interval, rather than a precise probability.

But which interval? $c(L_1) = [0, 1]$? Or $c(L_1) = [\frac{1}{4}, \frac{1}{2}]$, or perhaps even not an interval but just two values $c(L_1) = \{\frac{1}{2}, \frac{1}{4}\}$?

Tangent. Note that there are two key motivations for imprecise credences. The first, the one we've seen here, is that sometimes the evidence doesn't seem sufficient to determine a single probability, but rather only a range of probabilities. The second is that sharp probabilities are *psychologically unrealistic*: nobody literally has exactly a credence of 0.12472648572548... in a proposition. So perhaps a better model is to understand their credence as a range of values.

Though, if one's credence is $[x, y]$, we now have *two* precise values, not just one: x and y!

White's Coin Puzzle
White isn't satisfied with this response. Here's a famous puzzle he poses for proponents of imprecise credences.

> **Coin Game.** You haven't a clue as to whether p. But you know that I know whether p. I agree to write 'p' on one side of a fair coin, and not-p on the other, *with whichever one is true going on the heads side.* (I paint over the coin so you can't see which sides are *heads* and *tails*. We toss the coin and observe that it happens to land on p.

White notes that the following five claims are inconsistent, where c represents your initial credences, and c_+ represents your credences after seeing the coin land on p:

1. $c(p) = [0, 1]$
    - This is just the attitude that proponents of imprecise credences think I should have towards p when in a position of complete ignorance.
2. $c(heads) = \frac{1}{2}$

- You know the coin is fair. So that you should be $\frac{1}{2}$ confident it will land heads follows form the idea that your credences should match known objective chances.

This is the famous "Principal Principle" from Lewis (1980).

3. $c_+(p) = c_+(heads)$
   - You know the heads-side of the coin has the true proposition written on it. And you know it landed on the p-side. So you think it landed on heads if and only if p. So you should have the same credence in p and *heads*.

At this point, it already follows that on seeing the coin land on p, you either need to change your credence in p or change your credence in *heads*. The next two claims are that you shouldn't change your credence in p and that you shouldn't change your credence in *heads*, thus giving us a contradiction.

4. $C_+(p) = c(p)$
   - Suppose we think that, on seeing the coin land on p, you should sharpen your crednece in p to $\frac{1}{2}$. Then presumably the same should happen if we see the coin land on not-p. But then why wait for the coin to be flipped at all? Sharpen your crednece in *p now*.
5. $c_+(heads) = c(heads)$
   - Perhaps we instead deny 5, and think that your credence in *heads* should "dilate" from $\frac{1}{2}$ to $[0, 1]$? This is actually what the standard formal treatment of imprecise credences predicts. But White gives a series of objections:
     - **You'll violate Lewis's Principal Principle.** You *know* the coin is fair! So it looks like your second guessing things you know about the objective chances by being anything other than $\frac{1}{2}$ confident in *heads*.
     - **You'll violate van Fraassen's Reflection Principle.** Before flipping the coin, you know, no matter whether the coin lands p or not-p, that future you—who will have strictly more evidence—will have credence $[0, 1]$ in *heads*. But right now you're $\frac{1}{2}$ confident in heads. This seems irrational.

See Briggs (2009) for a nice introduction and detailed discussion of reflection principles

   The next two objections concern how having an imprecise credence should effect your behavior; in particular what bets you are willing to take. Suppose your credence in p is $[x, y]$—what bets should you be willing to take on whether p?

   Here are two suggestions:

   Liberal You are allowed to select any value within $[x, y]$ and make bets on p assuming it has that specific value.

   Conservative You are only allowed to bet on p is it's rational according to *every* value in $[x, y]$

- **Against Liberal.** One you see the coin land p, you assign $[0, 1]$ to *heads*. But then it's permissible to select the value $\frac{3}{4}$ for *heads*, and bet based on that. But now you'll be inclined to accept 2:1 bets on whether *heads*. You win 10 if heads and lose 20 if tails. If this keeps happening (I keep on writing p/not-p on fair coins, flipping them, and then offering you a 2:1 bet on heads) eventually you'll go broke!
- **Against Conservative.** This rules out foolish bets, but moreover rules out wise bets. Suppose you are offered the reverse bet: win 20 if heads, lose 10 if tails. This is not a wise bet according to all the values in $[0, 1]$. So Conservative tells you to turn this bet down.
  Meanwhile, your friend Sarah decided to close her eyes when the coin was flipped as so reamins $\frac{1}{2}$ confident in heads. Thus, she takes the bet. Suppose we keep on repeating this process for many coins. She closes her eyes and takes the bet on heads every time. On average she's winning $10 per bet. "Don't you want to get in on this?" she asks. "I can't," you reply. "I keep seeing how the coin lands, so none of these bets are rational for me"...

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