Credences Across Time: Cheat Sheet

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Remember the bare minimum synchronic constraints that are assumed for rational credences. All claims are assigned a number in the interval $[0, 1]$, and:

Normality: If p is a tautology, $C(p) = 1$

Finite Additivity: If p and q are mutual exclusive, then $C(p \text{ or } q) = C(p) + C(q)$

To understand how Bayesians tend to understand "diachronic" rationality—rationality across time, on gaining new information—we need to introduce some additional ideas:

- Let $C(p \mid q)$ represent your "conditional credences" in p: how confident are you in p, "if" or "given" or "supposing" q?[1]
- Let $C_n(p)$ represent your credence in p *at a specific time* t_n.[2]

Bayesians then often suggest the following constraints. Following Titlebaum, I'll interpret the following as merely proposing normative constraints—i.e. constraints on *rational* credences.

First we have:

Ratio Formula. When $C(q) > 0$, $C(p \mid q) = \frac{C(p\&q)}{C(q)}$

Drawing out a Venn diagram is useful for seeing what's so intuitive about this.

Two equations (that may be ***very*** useful for today) quickly follow from the Ratio Formula:[3]

Conjunction: $C(p\&q) = C(p \mid q)C(q)$

Independence: If p and q are probabilistically independent relative to C—that is, $C(p \mid q) = C(p)$—then: $C(p\&q) = C(p)C(q)$

Further, two extremely important equations that follow are:

Total Probability: $C(p) = C(p \mid q)C(q) + C(p \mid \neg q)C(\neg q)$.[4]

Bayes' Theorem: $c(p \mid e) = \frac{c(e|p)c(p)}{c(e)}$

So far, this is all synchronic— You have your conditional credences *at a time*. So, how should your credences change when you learn some information e?...

A very natural answer is: **your new credences, after learning e, should equal your old credences conditional on e...**

Conditionalization. Let e_2 represent the total evidence you've received between t_1 and t_2, then: $C_2(p) = C_1(p \mid e_2)$

[1] Titlebaum treats conditional credences as a genuine doxastic attitude. On this picture, we have "conditional" variants of most of our propositional attitudes (e.g. if Al hated the movie, I regret recommending it to him; if they have good vegan options, I want to have dinner at Au Lac) and the relationship between our categorical and conditional attitudes is a substantive matter rather than merely definitional.

[2] Credence functions are usually at least implicitly indexed to a time—it's just this is often unimportant and so the index isn't explicitly mentioned.

[3] These equations, along with the Ratio Formula, should also say they apply only when $C(q) > 0$ — I'm being sloppy by ignoring it.

[4] More generally, if $\{q_1, ...q_n\}$ is a partition, $C(p) = c(p \mid q_1)C(q_1) + ... + c(p \mid q_n)C(q_n)$

Some Additional Notes

- We've started out with unconditional credences and then moved onto conditional credences. It's therefore natural to think the latter should be understood in terms of the former. However:
  - It's often much, much easier to identify your conditional credence in something than your unconditional credence. Say you're 50/50 on whether this fair coin is $\frac{3}{5}$ biased towards heads or $\frac{7}{12}$ biased towards heads.
    - What's your credence it will land on heads on the next flip? Hmm....
    - What's your credence it will land on heads on the next flip conditional on it being $\frac{3}{5}$ biased towards heads? $\frac{3}{5}$!
  - It's also philosophically contentious which of unconditional versus conditional credences are "more basic". Al thinks conditional credences are more basic—you can ask him why!
- The issues we'll discuss today tempt some people to move away from this traditional Bayesian picture. But many others are resistant, as this Bayesian picture brings along many "Bayesian niceties". For example:[5]

[5] These results tend to assume that you always know what your evidence is, and always knows what your credences are. These "introspection" assumptions are hugely controversial, and a lot of recent research is about seeing what follows when these assumptions are given up.

Reflection/Deference It follows from this picture that you should defer to experts: if you know Colin has strictly more evidence as you do about p, and you know he's rationally assessed that evidence, on learning he's x confident that p, you should also be x confident that p.

Good's Theorem It follows from this picture—and standard expected value decision theory—that if you are offered the chance, before deciding whether to ϕ, to obtain cost-free evidence that bears on whether ϕ-ing is a good idea, you're rationally required too look at that evidence.

Dutch Books There are various theorems illustrating that, if you violate the above constraints (and if your credences correspond to your betting dispositions), you are susceptible to "dutch books": a series of bets, which you deem fair, that together guarantee you'll lose money.

Expected Accuracy There are various theorems showing that, if you violate the above constraints, you'll think that there are alternative credences you could have that, by your lights, are on average "closer to the truth". This suggests a kind of incoherence—why not change your credences to the ones you think are more accurate?